893. Phytolacca; fructu monopyreno, majore; folio longiore, glabro.

894. Pilofella, major, umbellifera, . macrocaulos. *Floribus eſt floſculoſis.* Col. Ecph. 248.

895. Roſa; Pimpinellæ folio, Scotica; flore eleganter variegato.
Roſa Ciphiana. Sibald. Scot. Illuſt.

896. Roſa ſylveſtris, Virginienſis.

897. Senecio Americanus; folio haſtato, nitide ſerrato.

898. Sium umbellatum, repens. Ger. Emac. 256.

899. Solanum, fruticoſum, Africanum; Lauri foliis.

900. Vulneraria erecta annua; folio ſubrotundo leviter crenato.
Loto affinis, Coryli folio. Dod. Mem.

VI. *A Phyſico-mathematical Demonſtration of the Impoſſibility and Inſufficiency of* Vortices: *By M.* de Sigorgne. *Tranſlated from the* French *by* T. S. *M. D.* F. R. S.

THAT natural Philoſophers of an inferior Claſs, who conſider only the Outſide of Things, are obſtinate in the Defence of *Vortices,* is, in my Opinion, not to be wondered at: The Idea of them ſtrikes the Mind very agreeably at firſt, and even ſeems to promiſe the true Mechaniſm. But that Perſons verſed in the moſt profound Geometry, and in the moſt ſublime Calculations, able Academicians, who inceſſantly apply themſelves to the Study of
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Nature, fhould plunge headlong into thefe Notions, and fuftain the *Vortices pro aris & focis*, is to me Matter of unaccountable Surprize.

It appears to me, that a *Vortex* is as fhocking upon mature Confideration, as its Idea is fatisfactory upon the firft mentioning it. And *Vortices*, in my Opinion, are like fmart Sayings *(bons mots)*, which charm the Mind the firft, or, perhaps, the fecond time, but by a Repetition become quite flat and infipid. What Man, indeed, (if free from Prejudice, and that the Spirit of Party has not depraved his Judgment) would not be aftonifhed to fee brought on the Scene, not only *Vortices*, but fuch as are compofed of an infinite Number of fmaller *Vortices*, each of which ftill contains an innumerable Number of others fubordinate to them? For Example : What is this Air, this Water, this Oil, &c. which Monfieur *de Moliers* takes pains to introduce? A Sport of the Imagination, or of the Mind, if you pleafe; but in reality a Paper-building. It has been long fince faid, that according as *Vortices* fhall be multiplied, they will degenerate into Littlenefs and Puerility: And thefe are the Sentiments even of the good *Cartefians* of our Days. But might it not be faid, that the great *Vortices* having the fame Origin with the little, the latter fhew the Meannefs of Extraction of the former? As Matter is divifible *in infinitum*; as to Vorticity, there is no Difference between the Great and the Small: And confequently, we have a Right to reject the large *Vortices*, fince *Cartefians* profcribe the fmall.

It is on this Confideration that I am refolved to attack the *Vortices:* For I muft own, to the Shame
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of our Nation, that the Spirit of Party is fo predominant therein, that feveral Perfons, who by a clofe Study have found the Infufficiency of *Vortices* for explaining the *Phænomena* of the Heavens, yet have not dared to publifh their Notions on that Subject.

But as at prefent the Syftem of fmall *Vortices* is freely attacked, I think, as already faid, that I have a Right to attack the large; and to this Purpofe I hope to prove,

I. That the mechanical Formation of a *Vortex* is impoffible.

II. That the *Vortex*, were it formed, cannot be of long Duration.

III. In fine, that it is not fufficient for explaining the *Phænomena*.

FIRST PART.

The mechanical Generation of the Vortex *is impoffible.*

DEMONSTRATION.

In the Hypothefis of a perfect *Plenum*, GOD at firft created Matter indefinite, uniform, homogeneous, and at Reft. This is allowed by all *Cartefians*, and follows in their Principles from this alone, that Matter was created at Reft. Now, from this perfect Homogeneity of Matter it evidently refults, in my Opinion, that the *Vortex* cannot be mechanically formed. Suppofe, fay the *Cartefians*, that while Matter is as yet at Reft, GOD imprints a Motion in a ftrait Line on one of its Particles: This Particle will every Inftant meet with Obftacles to the rectilinear

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linear Motion in the encompassing Matter; this Motion must therefore be turned aside, and will by this means become circular.

But why should the encompassing Matter, which is at Rest, be an Obstacle to the rectilinear Motion? Because, say they, it happens to be in the Line described by the Particle, on which Motion is supposed to be imprinted. But this very Reason would also prove, that the Body supposed to be in Motion could not circulate round a Centre at a Distance from it; because it would constantly meet with Matter at Rest in the Sides of the Polygon which it was to have described.

In a Word, it is a received Principle, that a Body which moves in a homogeneous Medium, never quits the Line of its first Direction: It does not refract, or deviate on one Side or the other of this Direction, except when it passes from an easier into a more difficult Medium, or from a denser into a less dense Medium: and even then its Direction must be oblique on the Surface of this Medium.

Now, the Body in Question would move in a Medium intirely homogeneous; seeing all the created Matter is supposed to be so, and that all but one Particle of this Matter is at Rest. It is moreover evident, that as all the Matter is uniform, every Direction, of what kind soever, of a Body which moves in the midst of this Matter, will be perpendicular to the Surface which corresponds to it; as is demonstrated in Mechanics. The supposed *Mobile* will therefore always move in the Line of its first Direction, until it has communicated all its Force; or rather it will

remain

remain at Reft after the leaft Shock, if Regard be had to nothing more than what I have hitherto faid.

But there ftill remains a very important Remark to be made on this Subject, to wit, that as it is univerfally agreed at this Day, that Reft is not a Force, all this Matter created at Reft will be infinitely foft: Its Parts will have no Tenacity, no Connexion, no Vifcofity; they will be but contiguous, and will not have more Adhefion to one another, than Two Globes which would touch out of the Bounds of the World without any reciprocal Attraction; fince Tenacity, Vifcofity, &c. are in the *Cartefian* Syftem but the Effects of Compreffion every Way. Wherefore thefe Parts will be divided at the leaft Shock, in the fame manner as if Quickfilver be thrown againft a Wall, it is inftantly feen to be divided into a Million of Parts, to be reflected on every Side, and be again divided as foon as it falls on the Floor. I know my Comparifon is not exact, but the Advantage is on my Side; becaufe Quickfilver is not without Vifcofity, or a certain Tenacity between its Parts; whether it proceeds from Attraction, which is my Opinion, or that it be the Effect of the Preffure of the ambient Fluid. Therefore the *Cartefian* Matter will have more Facility to divide than Quickfilver, and will not be fufceptible of any regular Motion; which alone demonftrates, that the mechanical Generation of the *Vortex* is impoffible.

There is however this Difference between the *Vortex* imagined by *Defcartes*, compofed of hard Globules; and that of the infinitely foft Matter of Father *Malebranche*, whofe Syftem is revived by his Difciple Monfieur *de Molieres*; that if the *Cartefians*

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admitted Gravity as a Principle; befides that it would give the true Caufe of Hardnefs, its Combination with the ftrait or projectile Motion would produce a Motion in a Curve ; as Sir *Ifaac Newton* has demonftrated. But until they will return to this Idea of primitive Gravity, and further while they will make ufe of no other Matter than one infinitely foft, and really unintelligible, it will not be poffible to conceive a fingle *Vortex* formed ; far from having this infinite Number, which, by-the-bye, ought to be diffipated as Waves raifed in the Water, upon account of their perfect Homogeneity.

The famous *Cartefians*, always refufing to allow this primitive Gravity, and at the fame time plainly feeing, that this firft Manner of forming the *Vortex* was impoffible, have had recourfe, in order to its Formation, to the Motion of Rotation of a folid Sphere at the Centre of a fmall Particle of Matter at Reft, *&c.* and they have pretended, that this Sphere in its Circulation ought to carry along with it the circumambient Matter.

But this Notion is certainly as unfuftainable as the Firft. For,

1*ft*, They muft explain to us the mechanical Formation of this Sphere; they muft account for its Solidity : But all this manifeftly fuppofes the *Vortex* already formed ; all this fuppofes a Preffure equal on every Side, uniform and concentric.

2*dly*, This Sphere would never imprint an equal Velocity on all the Points of the concave Surface which touches and inclofes it, feeing itfelf has not an equal Velocity in every Point of its laft Surface; and therefore the *Vortex* would not have as much

Force

Force to defend itfelf towards the Poles, as towards the Equator; as we fhall fhew hereafter.

3*dly*, This Sphere, in ftriking againft the ambient Matter, would but divide it *ad infinitum*; becaufe it is infinitely foft, and that its Parts have no Adherence with each other.

4*thly*, It is not fufficient, that a Sphere turns round its Centre, to draw into its Circulation the ambient Matter: It is moreover requifite, that to prefs on this Matter in a Direction from the Centre to the Circumference, (which a folid Globe either cannot do, or can hardly be conceived poffible for it to do) and further ftill, it is neceffary there fhould be Uneven-neffes on this Sphere, and on the concave Surface of the ambient Matter; becaufe otherwife, though the Sphere fhould prefs this Surface by its centrifugal Force, it would only raife it up, or tend to raife it, and it would flide along the Surface without dragging it away with it: On which Head there is this Particularity to be remarked, that, for the uniform Circulation and Confervation of the *Vortex*, and ftill more for the preferving of *Kepler*'s Laws, the Spheres and Surfaces muft be ftrictly Mathematical, as we fhall foon fee; and for its Formation they muft be rough, and full of Unevenneffes: But what can be more whimfical? And further, though thefe Surfaces were full of Prickles, yet could not the *Vortex* be formed in the Hypothefis of Father *Malebranche*'s foft Matter; becaufe the Parts which would form thefe Eminences and Unevenneffes on the concave Surface of the Matter furrounding the Sphere, not being connected with the other Parts of the fame Matter, would be carried off without Difficulty by the Rotation of

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the Sphere; and the reft of the Matter would remain at Reft. And thofe who would pretend, that thefe Unevenneffes, thefe Parts which form the Hillocks we are fpeaking of, could not, in confequence of God's Decree, loofe themfelves from the other Parts of the Matter, would evidently abandon Mechanifm, without reaping any Advantage: Becaufe, fuppofing it true, that by this Means the ambient Matter would be compelled to circulate, yet could it not form a fluid *Vortex*, wherein *Kepler*'s Laws could be obferved; becaufe both the Sphere and thefe Surfaces being by thefe Unevenneffes wedged into each other by folid hard and inflexible Parts, they would neceffarily move all of a Piece, as the Parts of a Sphere do.

5*thly*, By means of this Sphere one could have but a great *Vortex* formed; and not that infinite Multitude of fmall *Vortices*, with which the great ones are at this Day fuppofed to be filled, and in the Centre of all, or moft Part of which, People will not allow that there are hard Globules, and fo of the reft: For I am perfuaded, that the Reader, by a little Meditation on this Subject, will find almoft as many Reafons againft this Syftem, as there are fmall *Vortices* fuppofed to exift.

It may be objected, that we do not pretend to form a *Vortex*: We fuppofe that God formed it in the Beginning, and in Confequence hereof we account for its Properties and Confervation.

But, befides that the Impoffibility of the mechanical Generation of a *Vortex* is a ftrong Prejudice againft its Confervation; I pretend, in the Principles
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of our Adverſaries, God could not form a ſingle *Vortex*.

I deſire Attention may be given, that a circular Motion is a redoubled and forced Motion ; and not, as Mr. *Perault* thought, a natural Motion. Now the rectilinear Motion cannot be redoubled thus, as againſt its Nature, in order to become circular, but upon a Suppoſition that it meets in the ambient Matter invincible Obſtacles to its Direction ; or that by a primitive Law it is carried towards a Centre by a Motion of Gravitation, at the ſame time that it receives a Motion in a ſtrait Line. Therefore, ſince on one hand this univerſal and primordial Gravity is obſtinately rejected ; and on the other, as it is ſolidly proved above, that the ambient Matter is no Obſtacle to the rectilinear Motion ; it remains certain, that the Formation of the *Vortex* is impoſſible. *Q. E. D.*

Second *and* Third Part.

The Vortex, *though once formed, cannot laſt, and it is not ſufficient for explaining the celeſtial* Phænomena.

Postulatum.

The cylindric *Vortex* cannot long ſubſiſt, and is not ſufficient for explaining the celeſtial *Phænomena :* This Principle is allowed by all *Carteſians* in both its Parts. It cannot ſubſiſt ; becauſe not having Force to defend itſelf towards the Poles, if it happened to hit on that Side againſt another cylindric *Vortex*, that preſented its Equator, it would ſoon be broke into, and

and burft to its very Centre. If, on the contrary, its fame Side touched another cylindric *Vortex* by the Poles, they would both mix together, and would compofe but one *Vortex*.

It is not fufficient for explaining the celeftial *Phænomena*; becaufe it is allowed, that the tranflative Velocities of its Points cannot be in an inverted *Ratio* to the Roots of the Diftances, and that its centrifugal Force does not diminifh in the inverted *Ratio* of the Squares of thefe Diftances, *&c.*

COROLLARY.

Therefore the fpherical *Vortex*, in order to be of Ufe, muft have other Properties than the cylindric: That is to fay, it muft have a relative Force to one and the fame Centre; for it is by this Force alone that it can be different from the cylindric *Vortex*.

This Force, moreover, muft be equal in all the Points of the fame fpherical Superficies; becaufe otherwife it might be burft and broke into in its weak Parts, as well as the cylindric, *&c.*

THEOREM I.

Even in the fpherical *Vortex* there is no relative Force to one and the fame Centre: That is to fay, that it has properly but an axifugal Force.
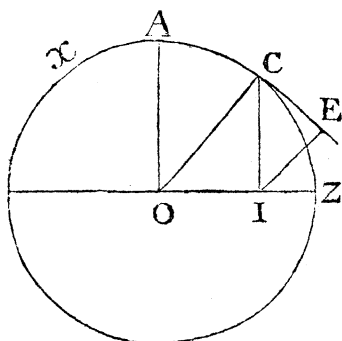
DEMONSTRATION.

The fpherical *Vortex* is compofed, as well as the cylindrical, of feveral parallel Circles, but with this Difference, that in the fpherical *Vortex* the *Radii* of the parallel Circles are not all equal, but on the contrary
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trary diminiſh according as they recede from the Equator, and approach the Poles. Now it is manifeſt, that all the parallel Circles circulating round different Points of the Axis in the ſpherical *Vortex*, as well as in the cylindrical, tend to recede only from theſe different Points of the Axis, round which they circulate; becauſe a Body cannot tend to recede from any Centre but that of its Circulation. In a Word, in order to make a *Vortex* ſpherical, which was cylindrical, they have but proportionally ſhortened the parallel Circles. But let the *Radius* of a Circle be ever ſo much ſhortened or lengthened, that will not change the Direction of its dilatative Effort. I am miſtaken! an imaginary Line is going to change the Direction of the axifugal Force. This Force, as all agree, has for its Direction the *Radius* I C, in the Circumference whereof it is the *Radius*; but the Direction I C is oblique to C E the Tangent to the Sphere; therefore it changes, according to the general Law of an oblique Shock, into the Determination I E or O C relative to the Centre O.

But if Lines may be imagined, and that nothing more is requiſite to realize them, than Points that correſpond to them; we ſhall have ſome of all ſorts in the *Vortex*: We ſhall have oblique Lines on the *Radius* O A, a perpendicular one, and ſome more or leſs oblique, on the *Radius* I C, and by that means we ſhall be able to determine nothing. Let us grant however, that there is a Tangent to the Sphere C E, at the Point C, and let us ſee if it will be a ſufficient Reaſon for decompoſing the centrifugal Force I C into a central Force I E or O C. For that Purpoſe I ask, What are the Points that compoſe this Tan-

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gent? It is evident, that it can only be the Globules of the upper *Stratum* that anfwer thereto. The Line C E is therefore compofed only of a certain Number of Points feparate one from the other, and which confequently can move one without the other. Therefore if the Line I C is perpendicular to the Globule that occupies the Point C, and that it paffes through its Centre; there will be no Decompofure, and the Force I C will not change into a Force that has the *Radius* O C for its Direction.

Now it is infinitely probable, that the *Radius* I C paffes through the Centre of the Globule C; and it is eafy to demonftrate, that it is actually fo even in the Principles of Monfieur *Saurin,* who firft invented this central Decompofition. For what has been the Caufe of the Decompofition of the circular Velocity into the centrifugal Force I C? It feems plain to me, that no other Caufe can be affigned than the Point or Globule C; feeing there is but that one at the Point where it happened. The Line I C paffes then through the Centre of the Globule C; fince the

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Decompofition is always made in a perpendicular Line to the Point that caufed it.

And indeed, either the *Radius* I C paffes through the Centre of the Globule C, or the Centre of this Globule is on one Side or the other of this *Radius*, but fo as that this *Radius* cuts the Point C; or elfe, it is a Space intercepted between Two Globules, which directly anfwers to the Point C. In the Firft Cafe, there is no Decompofition : In the Second, and in the Hypothefis, that the Centre of the Globule C happens to be between the *Radius* I C and the Equator, there will be a Decompofition; but it is manifeft, that it will not be a central one : It will, on the contrary, be relative either to the very Pole, or to one of the polar Circles. In the Third Cafe, wherein it is fuppofed, that it is a Space intercepted between Two Globules, which anfwers to the Point C; there may be a Decompofition, but it will be double, the one relative to the Centre O, and the other relative to the Pole Z.

Now the *Cartefians* can never draw from this Decompofition the Advantage they propofe; becaufe there will not be more Reafon for heavy Bodies precipitating to the Centre of the Sphere by means of the central Force, than to the very Pole by the Affiftance of the polifugal Force; or rather, the Complication of thefe Two Forces will compel the *Mobile* to precipitate to the Centre I of the Parallel it happens to be in.

Wherefore, in order to defend the fpherical *Vortex*, they muft fay, that the Centre of the Globule C is comprehended between the Poles and the *Radius* I C. But on what Foundation will they affure it?

What

What are the Proofs they will give for it? One muſt certainly be a very bold Gameſter, to hazard this Point; becauſe beſides the Appearance of Truth, the Adverſaries of *Vortexes* may wager Three to One, that it does not ſo happen. But in caſe it be allowed, will they ever find in the ſoft Matter of Father *Malebranche* and Monſieur *de Moliere*, a ſufficient Cauſe of the Decompoſition? There muſt be a Reſiſtance to produce a Decompoſition, and an infinitely ſoft Matter does not reſiſt. And further, in the Hypotheſis of the Decompoſition of I C or O C, the *Vortex* would not be in Safety; becauſe there would be a Remainder of the centrifugal Force I C, that would be parallel to the Tangent C E, and would evidently ſpread Confuſion in the *Vortex*, by driving all the parallel Circles towards the Equator.

This ſeems to me ſufficient to diſcredit, in the Minds of rational People free from Prejudice, this central Force, which is attempted by all means to be introduced. But let us not be tired of examining this Point thoroughly: It is of Conſequence, and the *Carteſians* well deſerve the Trouble of an abundant Refutation. Wherefore let us ſuppoſe, that G O D forms a *Vortex* cylindrical and fluid; it is a received and evident Principle, that its Points will have but an axifugal Force. And if a Sphere be conceived to be inſcribed in this Cylinder, the Points that compoſe it, will not in like manner have any central centrifugal Force, according to the Axiom: *Noſtrum intelligere nihil ponit in re.* Now let us realize this ſpherical *Vortex*, which before we had but conceived; that is, let us ſuppoſe, that G O D has deſtroyed the tranſlative Velocity of the Points that form the

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angular Spaces intercepted between the laft Surface of the infcribed Sphere and that of the Cylinder; it is manifeſt, that no Change will happen in the Velocity and axifugal Force of the reſt of the Points, which are not included in thefe; for this Reafon, that the Points which fill the Two kinds of Bafons that mark the Excefs of the Cylinder above the infcribed Sphere, remain in the fame Order, Difpofition, and Direction, with regard to the inferior Points, which they were in at the time of their Motion. And there is no other Difference to be perceived herein, except that at prefent it is the fame Point that conftantly correfponds to the fame Place; and that before this Place was fucceffively occupied by Points intirely refembling each other, and that which remains or is fuppofed conftantly to remain therein.

Now whether this Place be conftantly occupied by one and the fame Point, or fucceffively by Points intirely alike and in the fame Order, is what ought not to produce any Variation in the Effect which we are examining: And this appears to me at leaft as clear as Noon-day.

Wherefore, fince thefe inferior Points had then but an axifugal Force, it follows that even now they have no other Tendency than to recede from the Centres of their Circulations, without having any Force relative to the Centre of the *Vortex*.

This is all that pure Reafon dictates to me on this Point of the Nature of the *Vortex*, whether fpherical or cylindrical: And I dare flatter myfelf, that whofoever will attentively examine my Reafonings, will find

find them as demonftrative as can be defired in Natural Philofophy.

In Effect, Experience agrees here with Reafon. If a glafs Globe filled with Water be rapidly turned on its Axis, one fees little Foulneffes; the fmall Atoms which it never fails to contain, gather together along the Axis, and form a little Cylinder round it.—— Which very plainly fhews, that in this fpherical *Vortex* of Water there is but an axifugal Force. *Q. E. D.*

COROLLARY.

Therefore Gravity is inexplicable in the *Vortex,* and it has not Strength to defend itfelf towards the Poles.

THEOREM II.

Suppofing there was in the fpherical *Vortex* a central Force according to the *Radius* O C, it could not by Reaction be changed into a centripetal Force according to the *Radius* C O.

This Propofition is well known to all who are fomewhat converfant in Mechanics.

It is therein demonftrated, that if the *Radius* I C, for Example, forms with the Tangent C E an Angle of 45 Degrees, the Line of Reflexion will be parallel to the Axis; and that from the Point C to the Pole Z, the Lines of Reflexion will be divergent to the Axis; and, in fine, that from the Point C to the Equator, thefe fame Lines of Reflexion will be indeed convergent to the Axis, but will never terminate at the Centre O: In a word, that becaufe the Angle of Reflexion is always equal to the Angle
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of Incidence, it is only at the Equator that the centrifugal Force can be changed into a centripetal Force. *Q. E. D.*

COROLLARY I.

Therefore the modern *Cartefians* are ftrangely miftaken, when they pretend to account for Gravity by the Reverfe of the central centrifugal Force.

COROLLARY II.

And they can never, *à fortiori*, in their Principles, explain the Figure of the Earth and of *Jupiter*, which are flatted Spheroids made by the Converfion of an Ellipfis upon its fmall Axis.

LEMMA I.

If the centrifugal Force reprefented by I C (fee *p.* 420.) be decompofed on the fpherical Tangent into a Force, that for its Direction has the Centre of the Sphere; the central Force, which refults from this Decompofition, will be to the centrifugal Force, as the *Radius* I C to the *Radius* O C.

For the centrifugal Force I C, being decompofed into C on the Tangent of the Sphere, will ftrike this Tangent with a Force that will be reprefented by I E. But on account of the fimular Triangles I E C, I O C; I E. I C : : I C. O C.

LEMMA II.

A Body which defcribes a Curve, ftrikes this Curve every time it paffes from one Side to the other, with an infinitely fmall Force of the firft kind with regard to its Velocity.

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To the beſt of my Remembrance, this Propoſition is demonſtrated in Dr. *Clarke's* Notes on *Rohault's Phyſica*, and in Monſieur *de Moliere's* Lectures: And it is evident from this alone, that it can only be by a Force repreſented by the Sine of the Angle of Contact that this moveable Body ſtrikes the Tangent of its Curve.

THEOREM III.

Let us put Complaiſance on the Stretch, and grant that *Vortexes* have a central and centripetal Force relative to one Centre O: I ſay, that the ſpherical *Vortex* will not have as much of this central Force, to defend itſelf towards the Poles, as towards the Equator.

CONSTRUCTION.

Let us take, in the ſame Superficies X (ſee the Fig. *p.* 420.) Two Points at Pleaſure, the Point A in the Circumference of the Equator, and the Point C in the Circumference of a ſubduple parallel Circle; we will give in the Demonſtration an equal Velocity to the Globules which circulate in theſe Two Circumferences; which is the moſt favourable Conceſſion imaginable for the Patrons of *Vortexes*.

DEMONSTRATION.

It is manifeſt, that if the Point A is in an equal Space of Time ſtruck an equal Number of Times as the Point C, and that each Stroke againſt the Point A be double each Stroke againſt the Point C; it is manifeſt, I ſay, that there is more Force at the
<div align="right">Equator</div>

Equator than at the parallel Circle. Now the Suppofition is very certain in both its Parts: For,

1. Since the Circumference of the Equator is the double of that of the parallel Circle, and that being at an equal Diftance from the Centre O, (fee Fig. p. 420.) the Globules they contain are equal to each other; if there be a Thoufand Globules in the Circumference of the Parallel, there will be Two thoufand in the Circumference of the Equator. And as thefe Globules are fuppofed to have in both an equal Velocity, they will make (but) One Revolution in the Equator, while thofe of the fubduple Circumference will make Two. Therefore, in both, there will be Two thoufand Strokes employed in the fame Space of Time, againft the Points A and C.

2. Each central Stroke is double at the Equator: Becaufe, as there is in both an equal Velocity, and that (LEM. II.) each centrifugal Stroke in every Circumference is a Fluxion of the firft kind, with regard to the Velocity of the Globule which is in Motion; it follows that the centrifugal Strokes both in the Equator, and in the parallel Circle, are equal to each other. But the central Effort (which is the only one by which a *Vortex* can defend itfelf towards the Poles) is at the Point C (LEM. I.) but half the centrifugal Effort, fince it is reprefented by I E fubduple of I C; whereas at the Equator the central Effort is the fame with the centrifugal Effort, becaufe the *Radius* O A is perpendicular on the fpirical Tangent, which correfponds to it. Therefore, &c. Q.E.D.

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COROLLARY I.

Therefore if a *Vortex* be in *Æquilibrium* with another *Vortex*, and that the Equator of one happens to anfwer to the Poles or Tropics of the other; the latter will be burft and penetrated to the Centre: And I do not think, that the *Cartefians* can find their Account in this Confequence.

COROLLARY II.

Therefore if the *Vortex* was the mechanical Caufe of Gravity, Gravity ought to be greater at the Equator than at the Poles; and the Earth would be an oblong Spheroid; which is contrary to Obfervations.

REMARK.

I have faid, that it was making a large Conceffion to the *Cartefians*, to fuppofe that the Globules of both the Circumferences have an equal Velocity. For if a Sphere full of Water be made to turn on its Centre, Experience teaches, that the Velocity is greater at the Equator than in the parallel Circles; fince it is obferved, that the Times of their periodical Revolutions are equal. Whence it follows, that I have, in my Demonftration, made the moft favourable Suppofition for the *Cartefians* that was poffible.

THEOREM IV.

In order to determine the Tendency of a Layer towards the Upper Part of the *Vortex*, regard muft be had not only to that which refults from its own Circulation, but alfo to that which it receives from
the

the other lower Layers, unless it be the Layer next
the Centre.

DEMONSTRATION.

While a Layer is in Circulation, it visibly makes
a continual Effort towards dilating itself, by reason
of the centrifugal Force, with which all its Parts
endeavour to recede from the Centre of Circulation:
But its actual Dilatation being impeded by the Layer
next above it, this last will be naturally pressed by it.
And thus it is that the first or lowest Layer, being put
into Circulation, presses the Second; and the Second,
assisted by the First, presses the Third; this, assisted by
the Two preceding, presses the Fourth; and so on
from Layer to Layer, through the whole Extent of
the *Vortex*. Whence it follows, that in order to
estimate the Quantity of Force with which a Layer
tends towards the Surface of the *Vortex*, one must
take the centrifugal Force proper to this Layer and
that, which all the Matter of the Fluid contained under
it acquires by Circulation. *Q. E. D.*

COROLLARY I.

Therefore the dilatative Effort of the Layers in-
creases with the Layers in a greater Proportion than
these Layers.

COROLLARY II.

Therefore it is impossible to explain in the *Vortex*,
how Gravity decreases in an inverted *Ratio* of the
Squares of the Distances; and consequently there
will be nothing found in the *Vortex* to answer to

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Attraction, whofe Exiftence Sir *Ifaac Newton* has fo demonftratively eftablifhed.

COROLLARY III.

Thus we have re-eftablifhed in its full Light the Difficulty, which Monfieur *de Fontenelle* propofed to Monfieur *Villemot* in the Memoirs of the Academy for the Year 1705 *. This learned Academician pretends, that as in the *Vortex* the lower Points ought to move fafter than the upper, in order to preferve *Kepler's* Aftronomical Law; they ought alfo to have a greater centrifugal Force, and confequently compel them to defcend, particularly in proportion to their Fluidity. The Objection made a great Noife, and the only Method found of getting rid of it, was by faying, that although each lower Point had more centrifugal Force than each upper; yet as the *Vortex* was in *Æquilibrium*, and the Sums of the Force of each of the Two Layers were equal, there was no Reafon why the lower *Stratum* fhould get the better of the upper; becaufe this was as prevalent by the Number of its Points, as that was by the Force of each of its own.

But it is manifeft, after what has been demonftrated above, that the fecond Layer, being affifted by the firft, muft have a greater Force than the third, and confequently compel it to defcend, purfuant to the Principle then granted to Monfieur *Fontenelle*.

But if it be asked, How could the upper Layer defcend, feeing Matter is impenetrable?

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* He afterwards publifhed a Book, intituled, *Nouveau Syfteme*, ou *nouvelle Explication du Mouvement des* Planetes, *par M.* Philippe Villemot, *Pretre, Docteur en Theologie*, &c. Lyon, 1707. in 12°.

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I fhall ask in my turn, How, in an intire *Plenum*, do heavy Bodies fall to the Centre? And I reafon on the Principle granted to Monfieur *Fontenelle*.

But yet, becaufe what is allowed by one *Cartefian* is not always allowed by all; let us fuppofe, that the upper Layer cannot defcend; this, at leaft, will follow from my Demonftration, that, according to the Principles of all thefe Gentlemen, an upper Layer being preffed by all the under ones, it muft haften its Circulation, as long as it is flower than that of thefe under Layers; by reafon that the Excefs of their Velocities will act upon it, as if it had been at Reft.

COROLLARY IV.

Therefore the Layers of a *Vortex* will move all of a Piece, as do thofe of a folid Sphere; and *Kepler's* Law cannot poffibly be preferved. We fhall now give other Proofs upon other Principles.

THEOREM V.

The Motion of the Points of the Equator is abfolutely independent of the Motion of the parallel Circles; and confequently, in order to determine the *Æquilibrium* of the Points of the Equator, we muft attend to nothing but its Motion.

DEMONSTRATION.

The Plane of the Equator is parallel to the Planes of the other parallel Circles, that turn round the fame Axe with it: Its centrifugal Force is perpendicular to the Tangent to the Sphere, which anfwers to it: It has not then any lateral Tendency towards thefe

<div align="right">parallel</div>

parallel Circles, and by a neceffary Confequence its Motion is abfolutely independent of theirs.

And indeed, if it be fuppofed, that the Motion of the other parallel Circles ftops, there is ftill fome Motion conceived in the Equator, juft as in the Cafe of the cylindrical *Vortex :* It is likewife conceivable, that the Velocity may be greater at the Equator than in the parallel Circles, as the Experiment already cited fhews us: And if no Regard be had to the lateral Frictions, as the *Cartefians* would have it, who fuppofe them none or infenfible, and as indeed they are obliged to fay, that the *Vortex*, by the lateral Friction of the Equator, may not become cylindrical ; this Equator will always continue to circulate uniformly, without communicating any of its Velocity to the Points that laterally furround it. Therefore, *&c. Q. E. D.*

COROLLARY I.

Therefore for the *Æquilibrium* of the Points of the Equator, it is neceffary, at leaft, that an upper Circumference fhould have as much Tendency towards the Superficies of the *Vortex*, as another under concentric Circumference; becaufe, if it had lefs, there would be no *Æquilibrium*, even in the Principles of the *Cartefians* ; and the under Circumference, preffing the upper, would either make it defcend, or communicate to it a Force equal to its own. Wherefore, calling F the proper centrifugal Force of a Point of the upper Circumference, and f that of a Point of the under one ; if S, s mark the different Sums of the Points contained in thefe Two Circumferences, we fhall have $F S = f s$.

COROLLARY II.

Therefore the centrifugal Force does not diminish in the Plane of the Equator in the inverted *Ratio* of the Squares of the Distances from the Centre; for since $FS = fs$; $F.f :: s.S$. But the Points being supposed equal on both Sides, their Sums are as the Circumferences, and one has $s.S. :: d.\mathcal{D}$, which gives $F.f :: d.\mathcal{D}$. instead of $:: dd.\mathcal{D}\mathcal{D}$.

COROLLARY III.

Therefore *Kepler's* Rules cannot be observed in the *Vortex*, or at least in the Plane of its Equator; for since $F.f :: d.\mathcal{D}$; by putting in the Place of F, f, their Values, we shall have $\frac{VV}{D} . \frac{uu}{d} :: d.\mathcal{D}$, and therefore $V = u$ and $\mathcal{D}^2, d^2 :: TT.tt$. whereas we ought to have $V.u :: \sqrt[2]{d} . \sqrt[2]{D}$ and $\mathcal{D}^3. d^3 :: TT.tt$.

REMARK.

There is here a *Finesse* of the *Cartesians* to be observed. These Gentlemen consider only the *Æquilibrium* of the spherical Layers of the *Vortex*, and from the Equality of their central Forces they deduce *Kepler's* Laws, as well as they can.

But it is manifest, that whatever becomes of the Equality of Force in different spherical Superficies of the *Vortex*, there must be an *Æquilibrium* in the Plane of the Equator; because it is in this Plane that the Planets move; and if there had not actually been an *Æquilibrium* between its Points, they would soon place themselves there, by reason that Fluids always

always tend to the Side where they are lefs preffed; and it is by an actual *Æquilibrium* alone that they are kept in their Places; which intirely overturns the Theory of thefe Gentlemen.

Let us however grant to the *Cartefians*, that the Sums of the Forces of the Two fpherical Surfaces are equal; I cannot fee, that they can thence infer, as they do, that the central Force diminifhes in a reciprocal *Ratio* of the Square of the Diftance from the Centre. Let us examine their Argument:

$F\mathcal{S} = fs$, fay they; therefore $F.f :: s.\mathcal{S}$; but $s, \mathcal{S}$ mark the Sums of the Points contained in the Two Surfaces; therefore they are as thefe Surfaces, which, being as the Squares of their Diftances, give, $F.f :: dd.\mathcal{D}\mathcal{D}$.

But it muft be remarked, that the Surfaces of the *Vortex* are not Mathematical, they are Surfaces which have fome Thicknefs: They cannot then be proportional to the Squares of their Diftances from the Centre, except in the Cafe when their Thicknefs is equal. Now, according to the *Cartefians*, the Points or Globules, which compofe the *Vortex*, increafe in Bulk according as they recede from the Centre; and, befides, they are homogeneous, or of an equal fpecific Denfity, at leaft in their common Syftem. And confequently it is certain, that the different natural or real *Strata* of the *Vortex* are not of an equal Thicknefs, and that the Matter contained therein is not proportionate to the Squares of the *Radii* of thefe Surfaces, but only to the Squares of thefe *Radii* multiplied by the Thicknefs of the *Strata*. Therefore, &c. *Q. E. D.*

COROLLARY IV.

Therefore, even allowing the *Cartefians*, what one has a Right (COROL. I. THEOR. IV.) to refufe them, they will never be able to explain *Kepler's* Rules in the *Vortex*; for it is only by the Proportion, which I have juft now annulled, that they pretend to do it. See M. *de Molieres's Leçons de Phyfique*.

And if it be objected, that I have not, in the preceding Corollaries, had any regard to the Thicknefs of the Circumferences; I anfwer, that it was by way of pure Conceffion that I have not done it; and if any Perfon will be at the Pains of doing it, he will eafily find, that *Kepler's* Rules will only be the more difturbed thereby.

CONCLUSION.

Therefore the *Vortex* is every way impoffible, and infufficient in Natural Philofophy. Its mechanical Generation is impoffible (PART I.); it has only an axifugal Force, and not a centrifugal and centripetal Force, as it fhould have (THEOR. I. and II.); and even if it had, it cannot (THEOR. III.) defend itfelf equally on all Sides. It is not fufficient for explaining Gravity, and its Properties; it deftroys *Kepler's* Aftronomical Laws (COROL. III. THEOR. IV. and V.). What more can be defired, in order to conclude with Sir *Ifaac Newton?* " Itaque hypothefis Vorticum " (eft impoffibile &) cum phænomenis aftronomicis " omnino pugnat, & non tam ad explicandos quam " ad perturbandos motus cœleftes conducit." *Q.E.D.*

VII. *An*